Simplify $\binom{3n-2}{3n-5}$.

$$= \left| \frac{(3n-2)!}{(3n-5)!\,3!} \right| = \frac{(3n-2)(3n-3)(3n-4)(3n-5)!}{(3n-5)!\,3\cdot2\cdot1}$$

with $n-1$ written above $(3n-3)$

$$= \frac{1}{2}(3n-2)(n-1)(3n-4)$$

Find a_n for the geometric sequence with $a_3 = -\dfrac{12}{x^2 y}$ and $a_6 = \dfrac{3y^{11}}{2x^8}$.

$$a_n = a_1 r^{n-1}$$

$$a_3 = a_1 r^2 = \frac{-12}{x^2 y}$$

$$a_6 = a_1 r^5 = \frac{3y^{11}}{2x^8}$$

$$\frac{a_1 r^5}{a_1 r^2} = \frac{\dfrac{3y^{11}}{2x^8}}{\dfrac{-12}{x^2 y}} \quad 3$$

$$r^3 = -\frac{1}{8} \frac{y^{12}}{x^6} \quad 4$$

$$r = -\frac{y^4}{2x^2} \quad 3$$

$$a_1 \left(-\frac{y^4}{2x^2} \right)^2 = \frac{-12}{x^2 y} \quad 3$$

$$a_1 = -\frac{12}{x^2 y} \left(-\frac{2x^2}{y^4} \right)^2$$

$$= -\frac{12}{x^2 y} \cdot \frac{4x^4}{y^8}$$

$$= -\frac{48x^2}{y^9} \quad 4$$

$$a_n = -\frac{48x^2}{y^9} \left(-\frac{y^4}{2x^2} \right)^{n-1} \quad 3$$

Use sigma notation to write the series $-9 + 25 - 49 + 81 - \cdots - 961$.

$$-3^2 + 5^2 - 7^2 + 9^2 - \cdots - 31^2$$

$3, 5, 7, 9, \ldots, 31$

ARITHMETIC SEQUENCE

$a_n = 3 + 2(n-1) = 31$

$2(n-1) = 28$

$n - 1 = 14$

$n = 15$

$$\overset{3}{\underset{n=1}{\overset{15}{\sum}}} \underset{2}{(-1)^n} \underset{3}{\left(3 + 2(n-1)\right)^2}$$

$$= \sum_{n=1}^{15} (-1)^n (2n+1)^2$$

Consider the expression $(7x^{11} - 3x^5)^{21}$.   2 EACH EXCEPT AS NOTED   SCORE: _____ / 25 PTS

[a]   Write the first 3 terms of the expansion of the expression. Simplify all exponents.

Your answer may use multiplication and exponents, but **NOT** division, ! nor $_nC_r$ (or equivalent) notation.

$$(7x^{11})^{21} + \binom{21}{1}(7x^{11})^{20}(-3x^5) + \binom{21}{2}(7x^{11})^{19}(-3x^5)^2$$

$$= 7^{21}x^{231} - 21 \cdot 7^{20} \cdot 3x^{225} + \frac{21!}{2!19!} 7^{19} 3^2 x^{219}$$

$$\frac{21 \cdot \overset{10}{\cancel{20}} \cdot 19!}{\cancel{2} \cdot 1 \cdot 19!}$$

$$= 7^{21}x^{231} - 21 \cdot 7^{20} \cdot 3x^{225} + 21 \cdot 10 \cdot 7^{19} \cdot 3^2 x^{219}$$

[b]   Find the coefficient of x^{159} in the expansion.

Your answer may use multiplication, division, exponents and !, but **NOT** $_nC_r$ (or equivalent) notation.

$$\binom{21}{r}(7x^{11})^{21-r}(-3x^5)^r = \binom{21}{r} 7^{21-r}(-3)^r x^{11(21-r)+5r}$$

$$\text{COEFFICIENT} = \binom{21}{12} 7^9 (-3)^{12} = \frac{21!}{12!9!} 7^9 3^{12}$$

$$11(21-r)+5r = 159$$
$$231 - 6r = 159$$
$$-6r = 72$$
$$r = 12$$

Prove that $\displaystyle\sum_{i=1}^{n}[(i+1)\cdot(i+1)!-1]=(n+2)!-n-2$ for all positive integers n **using mathematical induction**. SCORE: _____ / 25 PTS

BASIS:
CASE $\displaystyle\sum_{i=1}^{1}\left[(i+1)\cdot(i+1)!-1\right]=2\cdot2!-1=3 \qquad 3!-1-2=6-1-2=3$

INDUCTIVE: ASSUME $\displaystyle\sum_{i=1}^{k}[(i+1)\cdot(i+1)!-1]=(k+2)!-k-2$ FOR SOME
STEP
$\qquad\qquad\qquad\qquad\qquad\qquad$ ARBITRARY INTEGER $k\geq1$

$\displaystyle\sum_{i=1}^{k+1}\left[(i+1)\cdot(i+1)!-1\right]$

2 EACH EXCEPT
AS NOTED

$=\displaystyle\sum_{i=1}^{k}[(i+1)\cdot(i+1)!-1]+(k+2)\cdot(k+2)!-1$

$=(k+2)!-k-2+(k+2)\cdot(k+2)!-1$

$=(1+k+2)\cdot(k+2)!-k-3$

$=(k+3)\cdot(k+2)!-(k+1)-2$

$=(k+3)!-(k+1)-2$

$=((k+1)+2)!-(k+1)-2$

SO, BY MI,
$\displaystyle\sum_{i=1}^{n}[(i+1)(i+1)!-1]$
$=(n+2)!-n-2$ FOR ALL
POSITIVE INTEGERS n

Find the value of $\sum\limits_{n=2}^{\infty} 600(0.5)^{4n-6}$. **HINT: Write out the first few terms first.**

$= 600(0.5)^2 + 600(0.5)^6 + 600(0.5)^{10} + \cdots$

$$= \dfrac{\overset{3}{600\left(\frac{1}{2}\right)^2}}{1 - \frac{1}{16}} \Bigg]^4$$

INFINITE GEOMETRIC SERIES

$r = (0.5)^4 = \frac{1}{16}$

$= \dfrac{150}{\frac{15}{16}} = \overset{10}{150} \cdot \frac{16}{15} = \underline{160}$

Consider the sequence defined recursively by $a_n = \frac{7}{3}a_{n-1} + 2a_{n-2}$, $a_1 = -\frac{9}{8}$, $a_2 = \frac{3}{4}$.

[a]   Find the first 6 terms of the sequence. **Your answers must be integers or fractions, NOT decimal approximations.**

$a_1 = -\frac{9}{8}$, $a_2 = \frac{3}{4}$

$a_3 = \frac{7}{3}a_2 + 2a_1 = \frac{7}{3}\left(\frac{3}{4}\right) + 2\left(-\frac{9}{8}\right) = \frac{7}{4} - \frac{9}{4} = -\frac{1}{2}$, $3\frac{1}{2}$

$a_4 = \frac{7}{3}a_3 + 2a_2 = \frac{7}{3}\left(-\frac{1}{2}\right) + 2\left(\frac{3}{4}\right) = -\frac{7}{6} + \frac{3}{2} = -\frac{7}{6} + \frac{9}{6} = \frac{1}{3}$, $3\frac{1}{2}$

$a_5 = \frac{7}{3}a_4 + 2a_3 = \frac{7}{3}\left(\frac{1}{3}\right) + 2\left(-\frac{1}{2}\right) = \frac{7}{9} - 1 = \frac{-2}{9}$, $3\frac{1}{2}$

$a_6 = \frac{7}{3}a_5 + 2a_4 = \frac{7}{3}\left(-\frac{2}{9}\right) + 2\left(\frac{1}{3}\right) = -\frac{14}{27} + \frac{2}{3} = -\frac{14}{27} + \frac{18}{27} = \frac{4}{27}$, $3\frac{1}{2}$

[b]   Based on the first 6 terms, does the sequence appear to be arithmetic, geometric or neither ? Show how you reached your conclusion.

GEOMETRIC, 2

$\frac{3}{4} / -\frac{9}{8} = \frac{3}{4} \cdot \frac{-8}{9} = -\frac{2}{3}$

$\frac{3}{4} \cdot \frac{-2}{3} = -\frac{1}{2}$

$-\frac{1}{2} \cdot \frac{-2}{3} = \frac{1}{3}$

$\frac{1}{3} \cdot \frac{-2}{3} = \frac{-2}{9}$

$-\frac{2}{9} \cdot \frac{-2}{3} = \frac{4}{27}$

4

QJ got a new credit card on December 1, 2015, and charged $47 on it that day. On the 1st day of every month   SCORE: _____ / 15 PTS
after that, QJ charged 1.3% more than she had charged on the 1st day of the previous month. By December 1, 2017, how much had QJ
charged on her card altogether ? (Assume that QJ never charged anything else to her card except on the 1st day of each month.)

$47 + 47(1.013) + 47(1.013)^2 + \ldots + 47(1.013)^{25-1}$ GEOMETRIC SERIES

$= \dfrac{47(1.013^{25} - 1)}{1.013 - 1}$

$= 1377.95$ DOLLARS